Test 2 - MTH-1400 Online
Dr. Adam Graham-Squire, Summer 2018

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____
(signature)

DIRECTIONS

1. Don't panic.

2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.

3. Clearly indicate your answer by putting a box around it.

4. Cell phones, computers, notes and textbooks are not allowed on this test. Calculators are not allowed on the first 4 questions of the test. Calculators are allowed on the last 7 questions, however you should still show all of your work. You will initially receive the entire test, and you will NOT be allowed a calculator. Once you have finished everything you can without a calculator, you should turn in the first part of the test (the first 4 questions) to the proctor. The proctor can then give you your calculator and you can finish the remaining questions. You are not allowed to go back to the No Calculator portion once you have been given your calculator.

5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.

6. Note that for some questions your calculator will need to be in degrees mode, and others it will need to be in radians.

7. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

8. If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.

9. Make sure you sign the pledge.

10. Number of questions = 11. Total Points = 60.

1. (5 points) (a) Convert the angle $\dfrac{4\pi}{3}$ from radians to degrees.

   (b) Find a *negative* angle (in degrees) that is coterminal with the angle $\dfrac{4\pi}{3}$.

   (c) Find a *positive* angle (in radians) that is coterminal with $\dfrac{4\pi}{3}$ (Note: I am looking for some angle *other* than $\dfrac{4\pi}{3}$).

2. (10 points) Find the following. You can use the unit circle on the next page to help you calculate the given trigonometric values, if you want, but filling in the unit circle itself will get you no points. If an expression is undefined, write DNE and briefly explain why it does not exist.

(a) $\sin\left(\dfrac{\pi}{6}\right) = $ _____

(b) $\tan\left(\dfrac{3\pi}{4}\right) = $ _____

(c) $\sec\left(\dfrac{\pi}{2}\right) = $ _____

(d) $\cot\left(\dfrac{7\pi}{2}\right) = $ _____

(e) $\cos\left(\dfrac{-\pi}{3}\right) = $ _____

(f) $\csc\left(\dfrac{4\pi}{3}\right) = $ _____

(g) $\arcsin\left(\dfrac{\sqrt{3}}{2}\right) = $ _____

(h) $\tan^{-1}(-1) = $ _____

(i) $\arccos\left(\sqrt{3}\right) = $ _____

(j) $\sin(-930°) = $ _____

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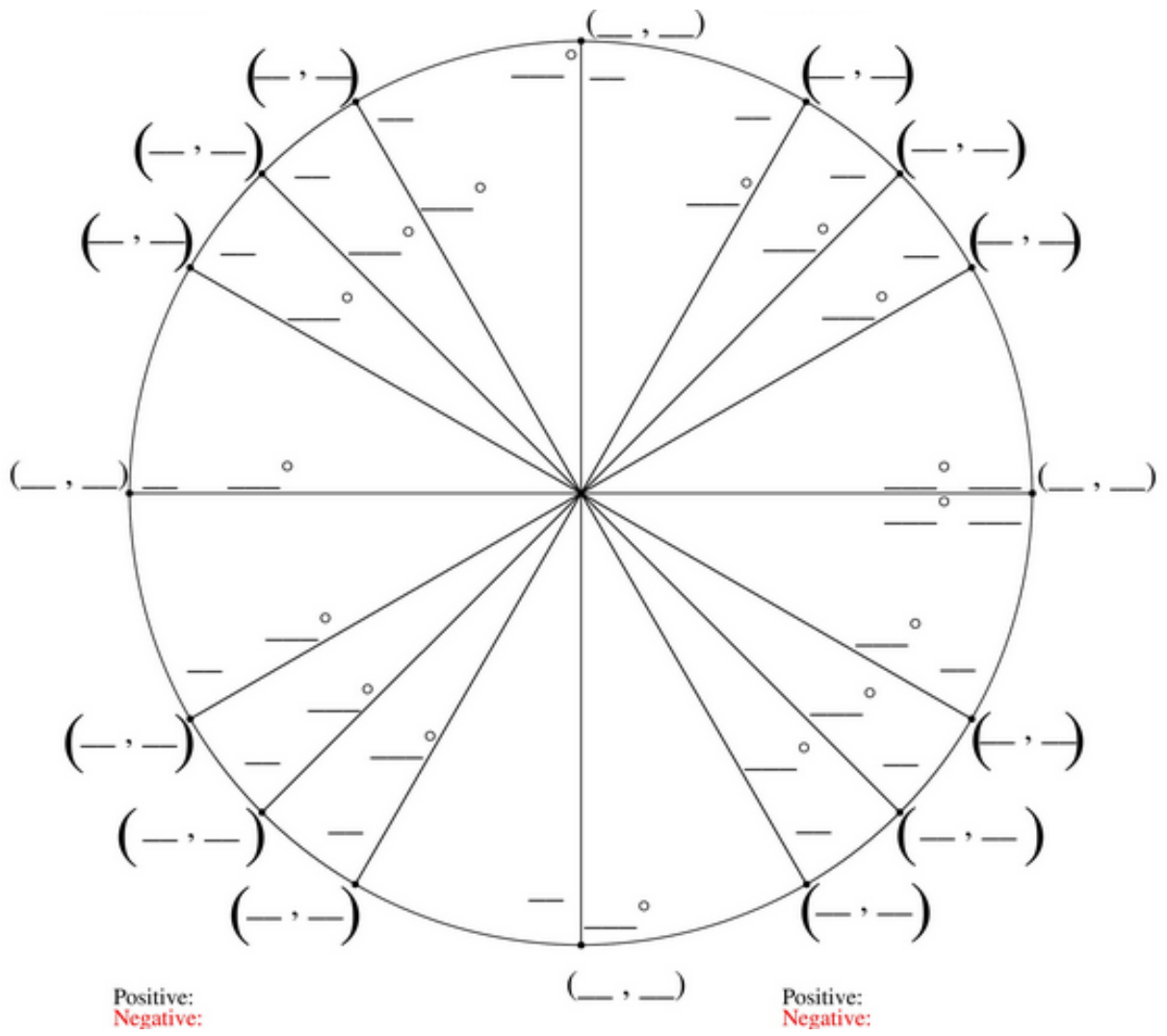
3. (5 points) (a) Starting with the identity $\sin^2 \theta + \cos^2 \theta = 1$, divide both sides of the equation by $\cos^2 \theta$ to find the trigonometric identity relating the values of $\tan \theta$ and $\sec \theta$.
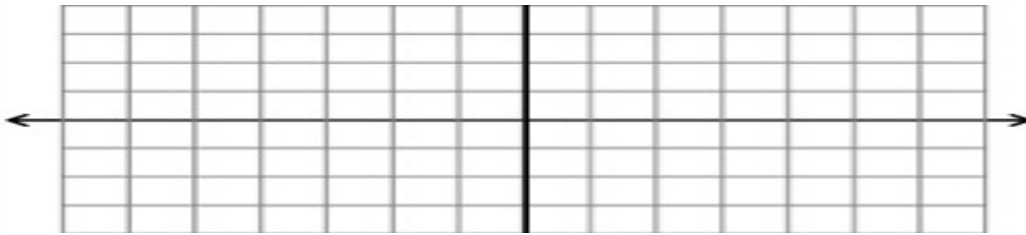
(b) If $\sin \theta = \frac{2}{3}$ and θ is in the second quadrant, use trigonometric identities to find the value of (i) $\cos \theta$, (ii) $\sec \theta$, and (iii) $\tan \theta$.

4. (5 points) (a) On the graphs below, sketch the graph for $y = \cos x$ and $y = \sin x$ (make sure to label some x-intercepts on each graph).

$y = \cos x$:
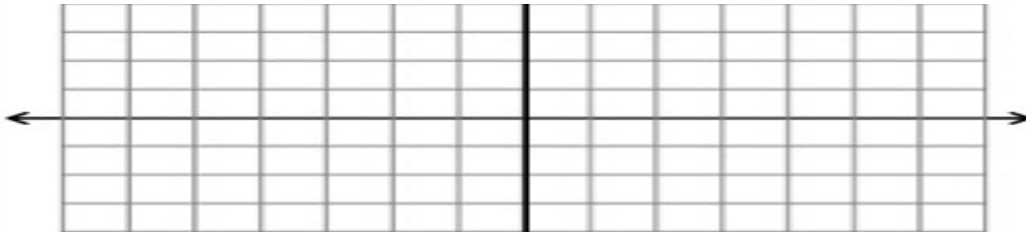
$y = \sin x$:

(b) Use the graphs to explain (in words) why the trigonometric identity

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$$

is true (if you cannot use the graphs above, you can also try to explain it using the unit circle, though that may be more difficult).

●Note that once you get your calculator and turn in the No Calculator portion, you CANNOT return to that part of the test!

●If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.

●If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

5. (5 points) Solve the equation

$$\log_{10}(x + 2) + \log_{10}(x - 1) = 1$$

either algebraically or graphically.

Whichever method you choose, you should explain/show your work.

6. (5 points) In 2009, there were approximately 18 million Twitter users. In 2011, there were 117 million Twitter users.

(a) Use the equation for modeling exponential growth to estimate the number of Twitter users there were in 2016.

(b) The *actual* number of Twitter users in 2016 was 319 million. How close to the actual value was your estimate from part (a)? If it was not close, briefly explain why you think the estimate was not accurate.

7. (5 points) Consider the triangle below, where angle $A = 75°$, angle B is a right angle, and side $\overline{BC} = 120$. Find the values of angle C, and sides $\overline{AB}$ and $\overline{AC}$. Round your answers to the nearest 0.1.

A

B   C
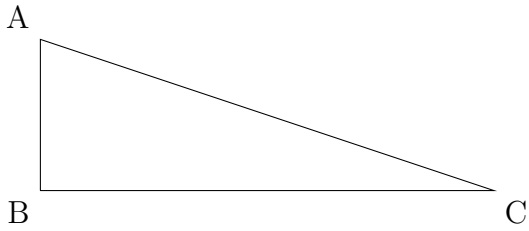
8. (5 points) A 108-foot tree casts a shadow that is 144 feet long. What is the angle of elevation (in degrees) of the sun? Round your answer to the nearest 0.01.

9. (5 points) For each of the given triangles ABC, determine if there is one, multiple, or no ways to solve the triangle. If there is one way to solve the triangle, find the remaining sides and angles. If there is more than one way to solve the triangle, solve *one* of the possible triangles, and explain how you would solve the other triangle (that is, what would be different).

Round answers to the nearest 0.1, and explain your reasoning/show your work where appropriate.

(a) $\angle A = 41.6°$, $a = 168.4$, and $b = 246.8$

(b) $\angle A = 43.7°$, $a = 112.9$, and $b = 209.3$

10. (5 points) On a day when the sun travels directly overhead at noon, a 5-foot tall woman casts a shadow of length

$$D(t) = 5 \left| \cot \frac{\pi}{12} t \right|$$

where the $|\,|$ symbols mean absolute value, D is measured in feet, and t is the number of hours that have passed since 6 AM. Note: your calculator must be in radians mode for this function.

(a) (i) How long is the shadow at 7 AM? (ii) How long is the shadow at 2 PM?

(b) At what time(s) of the day is the woman's shadow the same length as her height? Explain how to find the answer to this question *without* using a calculator (though you can use a calculator to check your work).

(c) What happens to the woman's shadow as the day gets closer to 6 pm? Explain why the function D gives this result.

11. (5 points) Explain, using your knowledge of the graph of $y = \sin x$, why the inverse sine function ($\arcsin x$) is only defined for x-values in the domain $[-1, 1]$ and has a range of $[\frac{-\pi}{2}, \frac{\pi}{2}]$. It is also fine to use the unit circle to support your argument.

Extra Credit: (2 points) From a satellite 600 miles above the earth, it is observed that the angle θ formed by the vertical and line of sight to the horizon is 60.276°. Explain how to use this information to find the radius of the earth. You do not actually have to find the radius, just explain how you would use math to do it.